

## **BRANS-DICKE SCALAR-TENSOR THEORY OF GRAVITATION MAY EXPLAIN TIME ASYMMETRY OF PHYSICAL PROCESSES**

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Most fundamental physical equations remain valid if we reversed the time order. Thus, if we start with a physical process (which satisfies these equations) and reverse time order, the resulting process also satisfies all the equations and thus, should also be physically reasonable. In practice, however, many physical processes are not reversible: e.g., a cup can break into pieces, but the pieces cannot magically get together and become a whole cup. In this paper, we show that the Brans-Dicke Scalar-Tensor Theory of Gravitation, one of the most widely used generalizations of Einstein's General relativity, is, in effect, time-asymmetric. This time-asymmetry may explain the observed time asymmetry of physical phenomena.

### **1. Time Asymmetry: Formulation of the Problem**

**Observable time asymmetry: a problem.** Most equations of fundamental physics are time symmetric, starting from the ordinary differential equations (e.g., the classical Newton's equations of motion) to partial differential equations describing physical fields like electromagnetism or gravitation. As a result, if we start with a physically reasonable solution to these equations (e.g., with the observed Universe) and simply reverse the direction of time  $t$ , the resulting fields will satisfy the same differential equations. From this theoretical viewpoint, all physical processes should be reversible: a time reversal of a physically reasonable process should also be physically reasonable.

In practice, however, many physical processes are not reversible. For example:

- If we drop a fragile cup, it will break into pieces.
- However, it is not physically reasonable to expect that the pieces of a broken cup would magically get together to form a whole cup.

**How this problem is explained now.** The problem of time asymmetry is known since Boltzmann's 19th century work on statistical physics and its foundations. In modern physics, this problem is usually resolved by making an additional assumption: that the initial conditions should be *random* (in some reasonable sense); see, e.g., [1].

**Limitations of the known explanation.** The additional assumption of randomness is outside the usual formulation of physical equations as a system of partial differential equations. It is therefore desirable to come up with an alternative explanation of the observed time asymmetry, an explanation that is within the usual formulation.

**What we do in this paper.** In this paper, we show that the time asymmetry problem can be potentially resolved if we take into account that for scalar-tensor theories of gravitation, in some reasonable sense, equations are *not* T-symmetric.

## 2. Brans-Dicke Theory Scalar-Tensor Theory of Gravitation: Reminder

*Notational comment.* Before we describe the actual equations, let us agree to simplifying notations which are commonly used in General Relativity.

The possibility of this simplification is based on the fact that, according to Relativity theory, the speed of light  $c$  is a universal constant. Thus:

- If we fix a unit of time (e.g., 1 sec), we automatically get a unit of length — namely, the distance that light can cover in 1 sec.
- Similarly, once we have a unit of length, we get a unit of time.

Relativistic equations can be simplified if we use units of distance and time which are related in this way, i.e., units in which the speed of light is simply equal to 1. For simplicity, we will use these units in our paper.

**General Relativity: reminder.** In general, the field equations of a physical theory correspond to the minimum of the action

$$S = \int L\sqrt{-g} dt dV,$$

where  $L$  is the Lagrangian and  $g = \det(g_{\alpha\beta})$  is the determinant of the metric tensor  $g_{\alpha\beta}$ . In other words, a physical theory corresponds to the variation principle

$$\delta \int L\sqrt{-g} dt dV = 0.$$

In particular, for the General Relativity theory (GRT), the Lagrangian has the form

$$L_{GRT} = \frac{1}{16\pi G} R + L_{\text{mat}},$$

where  $G$  is the gravitation constant,  $L_{\text{mat}}$  is the Lagrangian of matter,  $R \stackrel{\text{def}}{=} g^{\alpha\beta} R_{\alpha\beta}$  is the Ricci scalar,  $R_{\alpha\beta} \stackrel{\text{def}}{=} R_{\alpha\gamma\beta}^{\gamma}$ , and  $R_{\alpha\gamma\beta}^{\delta}$  is the curvature tensor. The variational equations do not change if we simply multiply the Lagrangian by a constant; it is therefore useful to multiply the Lagrangian by  $16\pi$  and take

$$L_{GRT} = \frac{1}{G} R + 16\pi L_{\text{mat}}.$$

Varying over  $g_{\alpha\beta}$ , we get the corresponding equations

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi G T_{\alpha\beta},$$

where  $T_{\alpha\beta}$  is the matter's energy-momentum tensor.

**Motivations for modifying General Relativity.** According to General Relativity, space-time and gravitation are described by Einstein's partial differential equations, in which the only field responsible for gravitation is the metric tensor  $g_{\alpha\beta}$ . In General Relativity — similarly to the original Newton's theory of gravitation — the gravitational field (and hence, gravitational acceleration  $a$ ) generated by a body of mass  $M$  at a distance  $r$  is proportional to its mass  $M$ , with the gravitation constant  $G$  as the proportionality coefficient:  $a \approx \frac{GM}{r^2}$ .

It turns out that the observed gravitational accelerations in the vicinity of several distant astronomical bodies are much larger than what is predicted based on the observable mass  $M_{\text{obs}}$ :  $a \gg \frac{GM}{r^2}$ . The traditional approach to this problem is to conclude that, in addition to the observable masses, there are also non-observable ones. In this approach, to explain the observations, we must assume that on the cosmological level, 95% of the mass is formed by hypothetical non-directly-observable “dark matter” and “dark energy”.

Some physicists argue that instead of introducing such hypothetical types of matter, it is more reasonable to conclude that the parameter  $G$  that described the local strength of gravitational interactions does not have to be a universal constant: measurements of  $G$  at different points in space-time can lead, in general, to different results. In effect, the values  $\varphi(x)$  of  $\frac{1}{G}$  measured at different space-time points  $x$  form a new scalar field. So, in such a theory, to describe the gravitational field, we need to present both the metric field  $g_{\alpha\beta}$  and the scalar field  $\varphi$ .

The corresponding scalar-tensor theory of gravitation was indeed proposed by Brans and Dicke (see, e.g., [2], Chapter 39); the equations of this theory are presented below.

*Historical comment.* Historically the first modification of General Relativity, in which there is no need for the hypothetical dark energy and dark matter, came in the form of a modified Lagrangian which only depends on the metric  $g_{\alpha\beta}$  but which — in contrast to the Lagrangian of General Relativity — is non-linear in the scalar curvature  $R$ . However, a recent paper [3] showed that such theories are equivalent to scalar-tensor theories of Brans-Dicke type.

**Brans-Dicke Theory: reminder.** In the scalar-tensor theory of gravitation, the parameter  $G$  that described the local strength of gravitational interactions is no longer a universal constant, it is equal to  $1/\varphi$ , where  $\varphi$  is the new scalar field. In terms of this new field, the Einstein's term  $\frac{1}{G}R$  from the Lagrangian takes the form  $\varphi R$ .

To get a full description of the scalar-tensor theory, we also need to add, to the Lagrangian, the term  $\frac{\varphi_{,\alpha}\varphi^{,\alpha}}{\varphi}$  describing the effective energy density of the scalar field. As a result, we arrive at the following Lagrangian:

$$L_{\text{BDT}} = \varphi \left( R - \frac{\varphi_{,\alpha}\varphi^{,\alpha}}{\varphi^2} \right) + 16\pi L_{\text{mat}}.$$

Varying over  $g_{\alpha\beta}$  and  $\varphi$ , we get the following equations:

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = \frac{8\pi}{\varphi}T_{\alpha\beta} + \frac{\omega}{\varphi^2} \left( \varphi_{,\alpha}\varphi_{,\beta} - \frac{1}{2}g_{\alpha\beta}\varphi_{,\gamma}\varphi^{,\gamma} \right) + \frac{1}{\varphi}(\varphi_{;\alpha\beta} - g_{\alpha\beta}\square\varphi); \quad (1)$$

$$\square\varphi = \varphi^{;\alpha}_{;\alpha} = \frac{8\pi}{3+2\omega}T, \quad (2)$$

where  $T \stackrel{\text{def}}{=} T^{\alpha}_{\alpha}$  is the trace of the energy-momentum tensor [2].

**At first glance, the Brans-Dicke theory is T-symmetric.** At first glance, from the viewpoint of time symmetry, the Brans-Dicke Theory (BDT) is similar to Einstein's General Relativity:

- similar to General Relativity, the Brans-Dicke Theory is described by second order partial differential equations, and
- the BDT equations remain invariant if we reserve the order of time  $t$ , i.e., change  $t$  to  $-t$ .

In general, in a second-order theory,

- if on some Cauchy surface (e.g., for some moment of time  $t_0$ ), we know the values of the gravity tensor  $g_{\alpha\beta}$ , the scalar field  $\varphi$ , and their first time derivatives  $\dot{g}_{\alpha\beta}$  and  $\dot{\varphi}$ ,
- then we can uniquely determine the second time derivatives  $\ddot{g}_{\alpha\beta}$  and  $\ddot{\varphi}$ , and thus (at least locally) integrate the corresponding equations.

### 3. Main Result: Cauchy Problem for the Brans-Dicke Scalar-Tensor Theory of Gravitation Leads to T-Asymmetry

**Our main result: formulation.** In this section, we prove a new (somewhat unexpected) result: that with respect to the scalar field  $\varphi$ , the Brans-Dicke scalar theory of gravitation is actually *first order*. Specifically:

- if on some Cauchy surface, we know the values of the gravity tensor  $g_{\alpha\beta}$ , its first time derivative  $\dot{g}_{\alpha\beta}$ , and the field  $\varphi$ ,
- then we can determine the first time derivative  $\dot{\varphi}$  of  $\varphi$  from a quadratic equation.

This quadratic equation, in general, has two solutions. This means that in principle, for each initial condition, we can have *two* different dynamics — corresponding to these two solutions.

**Discussion.** In physical terms, our result means that the time-symmetric Brans-Dicke Theory (BDT), in effect, consists of *two* different theories — each of which is second order in metric tensor and first order in  $\varphi$ .

Each solution of BDT is a solution of one of these two theories. In particular, our Universe satisfies one of the corresponding two systems of partial differential equations.

The transformation  $t \rightarrow -t$  transforms each of these two theories into another one, but none of these two theories is time-symmetric. In other words, in the presence of the additional scalar gravitational field, the equations describing our Universe are *not* time symmetric.

This may explain the observed time asymmetry of physical phenomena.

*Historical comment.* The fact that the first derivative  $\dot{\varphi}$  is not needed was first shown in Section 16.4 of [4] for homogeneous isotropic cosmological solutions. In this paper, we show that it is possible to describe  $\dot{\varphi}$  in terms of other initial conditions  $\varphi$ ,  $g_{\alpha\beta}$ , and  $\dot{g}_{\alpha\beta}$  in the general case as well.

**How we prove our result.** In the following text, we will transform the Brans-Dicke equations step-by-step. After these transformation, we will see that the first time derivative  $\dot{\varphi}$  of the scalar field  $\varphi$  on a Cauchy surface can indeed be determined in terms of the values of the metric field  $g_{\alpha\beta}$ , its first time derivative  $\dot{g}_{\alpha\beta}$ , and the field  $\varphi$  on this surface.

**First transformation: into equivalent equations for the Ricci tensor.** The equations (1) describe the Einstein tensor  $R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R$ . Let us first use these

equations to describe the Ricci tensor  $R_{\alpha\beta}$ . By definition,

$$R_{\alpha\beta} = \left( R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R \right) + \frac{1}{2}g_{\alpha\beta}R; \tag{3}$$

so, to describe the Ricci tensor, it is sufficient to be able to describe the Ricci scalar  $R$ . If we take the diagonal of the equation (1), we get

$$-R = \frac{8\pi}{\varphi}T - \frac{\omega}{\varphi^2}\varphi_{,\alpha}\varphi^{,\alpha} - \frac{3}{\varphi}\square\varphi,$$

so

$$R = -\frac{8\pi}{\varphi}T + \frac{\omega}{\varphi^2}\varphi_{,\alpha}\varphi^{,\alpha} + \frac{3}{\varphi}\square\varphi. \tag{4}$$

Substituting this expression (4) for  $R$  and the expression (1) for  $R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R$  into the formula (3), we get

$$\begin{aligned} R_{\alpha\beta} &= \frac{8\pi}{\varphi}T_{\alpha\beta} - \frac{4\pi}{\varphi}Tg_{\alpha\beta} + \frac{\omega}{\varphi^2} \left( \varphi_{,\alpha}\varphi_{,\beta} - \frac{1}{2}g_{\alpha\beta}\varphi_{,\gamma}\varphi^{,\gamma} \right) + \frac{1}{2}\frac{\omega}{\varphi^2}\varphi_{,\alpha}\varphi^{,\alpha} + \\ &\quad + \frac{1}{\varphi}(\varphi_{;\alpha\beta} - g_{\alpha\beta}\square\varphi) + \frac{3}{2\varphi}\square\varphi = \\ &= \frac{8\pi}{\varphi} \left( T_{\alpha\beta} - \frac{1}{2}Tg_{\alpha\beta} \right) + \omega\frac{\varphi_{,\alpha}\varphi_{,\beta}}{\varphi^2} + \frac{\varphi_{;\alpha\beta}}{\varphi} + \frac{1}{2}\frac{\square\varphi}{\varphi}g_{\alpha\beta}. \end{aligned}$$

Substituting the expression (2) for  $\square\varphi$  into this formula, we conclude that

$$R_{\alpha\beta} = \frac{8\pi}{\varphi} \left( T_{\alpha\beta} - \frac{1+\omega}{3+2\omega}Tg_{\alpha\beta} \right) + \omega\frac{\varphi_{,\alpha}\varphi_{,\beta}}{\varphi^2} + \frac{\varphi_{;\alpha\beta}}{\varphi}. \tag{5}$$

**Second transformation: into Gaussian normal coordinates.** We will use Gaussian normal coordinates, in which  $g_{00} = 1$  and  $g_{0i} = 0$  for  $i = 1, 2, 3$ , so only the values  $g_{ij}$  corresponding to  $i, j = 1, 2, 3$  are changing. In these coordinates, the distance element  $ds^2 = g_{\alpha\beta}dx^\alpha dx^\beta$  takes the form

$$ds^2 = dt^2 - \gamma_{ij}dx^i dx^j,$$

where we denoted  $\gamma_{ij} \stackrel{\text{def}}{=} -g_{ij}$ . In the Gaussian normal coordinates, the components  $R_{00}$ ,  $R_{0i}$ , and  $R_{ij}$  of the Ricci tensor  $R_{\alpha\beta}$  can be explicitly described in terms of  $\gamma_{ij}$ ,  $\varkappa_{ij} \stackrel{\text{def}}{=} \dot{g}_{ij} = -\dot{\gamma}_{ij}$ , and the time derivative  $\dot{\varkappa}_{ij}$ ; see, e.g., [2]. Substituting these expressions for  $R_{00}$ ,  $R_{0i}$ , and  $R_{ij}$  into the formula (5), and using similar expressions for the derivatives of  $\varphi$ , we get the following equations:

$$-\frac{1}{2}\dot{\varkappa}_i^i - \frac{1}{4}\varkappa_j^i \varkappa_i^j = \frac{8\pi}{\varphi} \left( T_{00} - \frac{1+\omega}{3+2\omega}T \right) + \omega\frac{(\dot{\varphi})^2}{\varphi^2} + \frac{\ddot{\varphi}}{\varphi}; \tag{6}$$

$$\frac{1}{2}\varkappa_{i;j}^j - \frac{1}{2}\varkappa_{j;i}^j = \frac{8\pi}{\varphi}T_{0i} + \omega\frac{\dot{\varphi}\varphi_{,i}}{\varphi^2} + \frac{\dot{\varphi}_{,i}}{\varphi}, \tag{7}$$

$$\begin{aligned}
 & P_{ij} - \frac{1}{2}\dot{\varkappa}_{ij} - \frac{1}{4}(\varkappa_{ij}\varkappa_k^k - 2\varkappa_i^k\varkappa_{kj}) = \\
 & = \frac{8\pi}{\varphi} \left( T_{ij} + \frac{1+\omega}{3+2\omega} T\gamma_{ij} \right) + \omega \frac{\varphi_{,i}\varphi_{,j}}{\varphi^2} + \frac{\varphi_{;ij} - \varkappa_{ij}\dot{\varphi}}{\varphi}, \tag{8}
 \end{aligned}$$

where  $P_{ij}$  is the 3-D curvature tensor, and all tensor operations are performed in the space with metric  $\gamma_{ij}$ .

The equation (2) takes the form

$$\ddot{\varphi} - \Delta\varphi = \frac{8\pi}{3+2\omega} T, \tag{9}$$

where  $\Delta\varphi \stackrel{\text{def}}{=} \varphi_{;i}^i$  is the 3-D Laplace operator.

**It is possible to describe  $\dot{\varphi}$  in terms of  $\gamma_{ij}$ ,  $\dot{\gamma}_{ij}$ , and  $\varphi$ : idea of the proof.**

The above equations describe the second time derivatives of  $\varphi$  and  $\gamma_{ij}$  in terms of the values  $\varphi$  and  $\gamma_{ij}$  and their first time derivatives.

Let us show that we can actually describe the *first* derivative  $\dot{\varphi}$  in terms of  $\gamma_{ij}$ ,  $\dot{\gamma}_{ij}$ , and  $\varphi$ . Indeed:

- from the equation (9), we can explicitly express  $\ddot{\varphi}$  in terms of  $\gamma_{ij}$ ,  $\dot{\gamma}_{ij}$ , and  $\varphi$ ;
- from the equation (8), we can explicitly express  $\dot{\varkappa}_{ij}$  (and, thus,  $\dot{\varkappa}_i^i$ ) in terms of  $\gamma_{ij}$ ,  $\dot{\gamma}_{ij}$ ,  $\varphi$ , and  $\dot{\varphi}$ ; the resulting dependence of  $\dot{\varkappa}_i^i$  on  $\dot{\varphi}$  is linear.

Substituting these expression for  $\dot{\varkappa}_i^i$  and  $\ddot{\varphi}$  into the equality (6), we get a quadratic equation for  $\dot{\varphi}$ . This quadratic equation allows us to determine  $\dot{\varphi}$  in terms of  $\gamma_{ij}$ ,  $\dot{\gamma}_{ij}$ , and  $\varphi$ .

Since the equation is quadratic, for each combination of initial values  $\gamma_{ij}$ ,  $\dot{\gamma}_{ij}$ , and  $\varphi$ , we may get *two* possible values of  $\dot{\varphi}$ .

**How to describe  $\dot{\varphi}$  in terms of  $\gamma_{ij}$ ,  $\dot{\gamma}_{ij}$ , and  $\varphi$ : details.** From the equation (9), we can conclude that

$$\ddot{\varphi} = \Delta\varphi + \frac{8\pi}{3+2\omega} T. \tag{10}$$

Similarly, from the equation (8), we conclude that

$$\begin{aligned}
 -\frac{1}{2}\dot{\varkappa}_{ij} & = -P_{ij} + \frac{1}{4}(\varkappa_{ij}\varkappa_k^k - 2\varkappa_i^k\varkappa_{kj}) + \frac{8\pi}{\varphi} \left( T_{ij} + \frac{1+\omega}{3+2\omega} T\gamma_{ij} \right) + \\
 & + \omega \frac{\varphi_{,i}\varphi_{,j}}{\varphi^2} + \frac{\varphi_{;ij} - \varkappa_{ij}\dot{\varphi}}{\varphi}, \tag{11}
 \end{aligned}$$

Hence, we get

$$-\frac{1}{2}\dot{\varkappa}_i^i = -P + \frac{1}{4} \left( (\varkappa_i^i)^2 - 2\varkappa_j^i\varkappa_i^j \right) + \frac{8\pi}{\varphi} \frac{6+5\omega}{3+2\omega} T + \omega \frac{\varphi_{,i}\varphi^{,i}}{\varphi^2} + \frac{\Delta\varphi - \varkappa_i^i\dot{\varphi}}{\varphi}. \tag{12}$$

Substituting formulas (10) and (12) into the equation (6) and moving all the terms to the right-hand side, we get the desired quadratic equation

$$A(\dot{\varphi})^2 + B\dot{\varphi} + C = 0, \tag{13}$$

where

$$\begin{aligned} A &\stackrel{\text{def}}{=} \frac{\omega}{\varphi^2}; \quad B \stackrel{\text{def}}{=} \frac{\varkappa_i^i}{\varphi}; \quad \text{and} \\ C &\stackrel{\text{def}}{=} \frac{1}{4} \varkappa_j^i \varkappa_i^j + \frac{8\pi}{\varphi} \left( T_{00} - \frac{1+\omega}{3+2\omega} T \right) + \frac{\Delta\varphi}{\varphi} + \frac{8\pi}{\varphi} \frac{1}{3+2\omega} T + P - \\ &\quad - \frac{1}{4} \left( (\varkappa_i^i)^2 - 2\varkappa_j^i \varkappa_i^j \right) - \frac{8\pi}{\varphi} \frac{6+5\omega}{3+2\omega} T - \omega \frac{\varphi_{,i} \varphi^{,i}}{\varphi^2} - \frac{\Delta\varphi}{\varphi} = \\ &\quad - \frac{8\pi}{\varphi} \left( T_{00} - \frac{6+6\omega}{3+2\omega} T \right) + P + \frac{3}{4} (\varkappa_i^i)^2 - \frac{1}{4} \varkappa_j^i \varkappa_i^j - \omega \frac{\varphi_{,i} \varphi^{,i}}{\varphi^2}. \end{aligned} \tag{15}$$

*Mathematical comment.* When the quadratic equation has two solutions  $\dot{\varphi}_1$  and  $\dot{\varphi}_2$  for  $\dot{\varphi}$ , we can substitute both solutions into the equation (7) and take the difference between the resulting equalities. Then, for the difference  $D \stackrel{\text{def}}{=} \dot{\varphi}_1 - \dot{\varphi}_2$ , we get the equality

$$\omega \frac{D\varphi_{,i}}{\varphi^2} + \frac{D_{,i}}{\varphi} = 0.$$

Multiplying both sides of this equation by  $\varphi$  and dividing by  $d$ , we conclude that

$$\omega \frac{\varphi_{,i}}{\varphi} + \frac{D_{,i}}{D} = 0,$$

i.e., equivalently,

$$\omega(\ln(\varphi))_{,i} + (\ln(D))_{,i} = (\omega \ln(\varphi) + \ln(D))_{,i} = 0.$$

Since the gradient of the expression  $\omega \ln(\varphi) + \ln(D)$  is equal to 0, this expression is constant, so  $\omega \ln(\varphi) + \ln(D) = c$ ,  $\ln(D) = c - \omega \ln(\varphi)$  and thus,

$$\dot{\varphi}_1 - \dot{\varphi}_2 = D = c_1 \cdot \varphi^{-\omega},$$

where we denoted  $c_1 \stackrel{\text{def}}{=} e^c$ .

**The same T-asymmetry holds for more general scalar-tensor theories of gravitation.** The authors of [3] also consider generalizations of Brans-Dicke theory, with the Lagrangian

$$L = \varphi \left( R - \frac{\varphi_{,\alpha} \varphi^{,\alpha}}{\varphi^2} - V(\varphi) \right) + 16\pi L_{\text{mat}},$$

where  $V(\varphi)$  is the potential of the scalar field. The original Brans-Dicke theory is a particular case of this general theory, corresponding to  $V(\varphi) = 0$ . For this



Lagrangian, the variational equations (generalizing equations (1) and (2)) take the form

$$R_{\alpha\beta} - \frac{1}{2}g_{\alpha\beta}R = \frac{8\pi}{\varphi}T_{\alpha\beta} + \frac{\omega}{\varphi^2} \left( \varphi_{,\alpha}\varphi_{,\beta} - \frac{1}{2}g_{\alpha\beta}\varphi_{,\gamma}\varphi^{,\gamma} \right) + \frac{1}{\varphi}(\varphi_{;\alpha\beta} - g_{\alpha\beta}\square\varphi) - \frac{1}{2}\frac{V(\varphi)}{\varphi}g_{\alpha\beta}; \quad (16)$$

$$\square\varphi = \varphi^{;\alpha}_{;\alpha} = \frac{8\pi}{3+2\omega}T - \frac{1}{3+2\omega} \left( V - \varphi\frac{dV}{d\varphi} \right). \quad (17)$$

The two additional terms depend only on  $\varphi$ , so, as one can see, they do not change the fact that the first derivative  $\dot{\varphi}$  can be (almost) uniquely determined by the initial values of  $\varphi$ ,  $g_{ij}$ , and  $\dot{g}_{ij}$ .

*Comment.* It is worth mentioning that a similar possibility to reconstruct  $\dot{\varphi}$  from the initial values of  $\varphi$ ,  $g_{ij}$ , and  $\dot{g}_{ij}$  is also available for more traditional scalar-tensor theories, with a Lagrangian of the type

$$L = \frac{1}{G}R + L_{\text{scalar}}(\varphi, \varphi_{\alpha}\varphi^{,\alpha}) + 16\pi L_{\text{mat}}.$$

For these theories, the proof of this reconstruction possibility is even easier than for the Brans-Dicke-type theories. Indeed, for such more traditional scalar-tensor theories, the right-hand side of the corresponding Einstein equations  $R_{\alpha\beta} - \frac{1}{2}Rg_{\alpha\beta} = \dots$  depends only on the first derivatives  $\dot{\varphi}$  and  $\varphi_{,i}$  of the scalar field. In particular, the right-hand side of the equation corresponding to  $R_{0i}$  (similar to our equation (7)) contains only  $\dot{\varphi}$  and thus, can be used to explicitly express  $\dot{\varphi}$  in terms of  $\varphi$ ,  $g_{ij}$  and  $\dot{g}_{ij}$ .

## 4. Conclusion

Our result shows that the time-symmetric Brans-Dicke Theory of gravitation (BDT), in effect, consists of two different theories — each of which is second order in metric tensor and first order in  $\varphi$ . Each solution of BDT is a solution of one of these two theories. In particular, our Universe satisfies one of the corresponding two systems of partial differential equations. The transformation  $t \rightarrow -t$  transforms each of these two theories into another one, but none of these two theories is time-symmetric.

In other words, in the presence of the additional scalar gravitational field, the equations describing our Universe are *not* time symmetric. This may explain the observed time asymmetry of physical phenomena.

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