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# ON APPLICATIONS OF LENNARD-JOHNES POTENTIAL AS A SOCIAL FORCE FOR CROWD BEHAVIOR MODELING

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**Abstract.** In this paper we provide the short overview of the currently existing approaches to modeling the crowd behavior. We consider the crowd as a set of particles moving under influence of physical, social and psychological forces. Starting from the Helbing's social forces model to describe the crowd motion we introduce Lennard-Johnes potential for modeling interaction force between individuals in the crowd. We show that Lennard-Johnes potential can account for the people's desire to preserve an optimal physical and social distance between each other; however, some modifications needs to be introduced in its calculations to account for directional anisotropy and other physical and behavioral aspects of motion of people within the crowd. In order to provide a performance evaluation of this model, we show a preliminary set of results obtained in a simulated scenario.

**Keywords:** social force model, evacuation modeling, crowd modeling, Lennard-Johnes potential, particle model.

## 1. Introduction

Crowd dynamics modeling is an open research topic that has attracted many investigations so far. The complexity of this problem stems from some objective difficulties in formalizing the human behavior, which dynamics are determined by parameters of physical, social and psychological nature that can't be accurately represented in their totality; their stochastic nature and complexity of verification, even with rich experimental statistics accumulated. On the other hand, crowd modeling has now become an important issue in many fields including architectural design, military simulation, safety engineering, computer-generated imagery etc.

To investigate the crowd dynamics researchers concentrate on different aspects of the crowd dynamics, e.g. movement patterns, handling external events, emerging effects in large crowds etc. Despite the fact that this research area is actively developing, only a small number of existing proposals takes into account all sides of crowd behavior in complex. In most existing models, only the physical and social interaction aspects of crowd motion a considered, neglecting to account for individual psychological variations, which make a significant contribution to the choice of strategy and tactics of the movement of individuals and overall crowd dynamics.

Many approaches and modeling methods are used to describe the crowd behavior. They can generally be divided into several classes:

- Flow-based: These models are based on an analogy with the continuous flow of fluid or gas and describe changing of density and velocity with time using partial differential equations such as Navier-Stokes or Bernoulli equations (Henderson, 1971; Bradley, 1993; Hughes, 2002 et al.).
- Particle-based approach to crowd modeling stems from the representation of the crowd as a set of homogeneous particles which motions in physical space are determined by the influence of various forces described by some global and local laws on the basis of Newtonian mechanics (Helbing, 1995; Bratsun, 2013 et al.).
- Agent based: each individual in a crowd is represented as an autonomous agent, which behavior is regulated by sets of properties and interaction rules. Each agent may take decisions independently (Braun, 2003; Pelechano, 2005 et al.).
- Cellular automata based: the moving field is represented as a finite regular grid of cells each in one has a finite number of states with the given transition rules. These rules determine a new state of each cell in terms of its current state and states of the cells in its neighborhood (Varas, 2007; Daoliang, 2006; Chertock, 2012 et al.).
- Mixed approaches that use a combination of several of the above modeling methods (Zheng, 2002; Guo, 2008 et al.).

We are going to consider the particle-based approach, outlined by the Helbing model, with some additions from agent-based models.

The social force model proposed by Helbing [1] has made a significant contribution in this area. In this model each individual in a crowd is represented as a particle with associated mass and velocity moving under influence of mixed physical and social forces. Some modifications of the model give a possibility to describe the crowd behavior in the state of panic during the evacuation in emergency situations such as fire [2].

In recent years the social forces models has received numerous modifications and extensions by other researchers [1, 3, 4].

Among the models of this class the work of Hoogendoorn and Bovy [5] should be noted. They investigated pedestrians behavior in the choice of routes and activity scheduling. The model was optimized by dynamic programming methods for different traffic conditions.

Another interesting model [6] was developed to describe short term behavior of individuals as a response to the presence of other pedestrians. Authors used a dynamic and individual-based spatial discretization, representing the physical space.

The model was calibrated using data from actual pedestrian movements, manually taken from video sequences, and produced good results.

Thus, we can summarize that particle-based models successfully simulate the most typical phenomena of crowd dynamics and achieve very realistic simulation results. The main problem of this approach is the necessity of homogeneity of the particles that in reality, of course, is not observed. Assigning individual parameters to particles in such models in most cases moves them into the agent-based or mixed classes that makes them much more difficult in analytic investigation.

## 2. Model description

### 2.1. Crowd behavior: determinants and principles of crowd movement

Crowd dynamics modeling makes it necessary to take into account a huge number of factors that affect the behavior of each individual in the crowd. First of all, these factors include physically measurable parameters, such as the motion space configuration, physical position and velocity of each participant.

Social factors, for example the presence or absence of social ties, social norms and cultural characteristics of individuals in the crowd are more complex to measure. These factors are more difficult to be formalized, but they make a significant contribution to the overall crowd dynamics, determining such phenomena as following the leader, grouping, queues formation, hustling etc.

Finally, the individual psychological characteristics of participants: their emotional state, assessment of the situation, decision-making, forecasting and planning are essential but very difficult to represent, especially analytically.

Also, it might be necessary to take into account the variety of participants' physical condition, e.g. their age, the presence of certain disabilities, as well as characteristics of the environment, such as visibility, presence of signs, etc. Taking these factors into consideration, we formulated the following principles describing the movement of each individual in the crowd:

1. Each individual tries to achieve a certain target can be a location (e.g. "Exit") or direction (e.g., "right"). Also, the target can be dynamic (e.g., "follow the leader" or "go to the nearest aisle").
2. Each individual tends to move with the most comfortable velocity which depends on the psychological evaluation of the nearest space and the individual state.
3. Every individual makes a decision about his future movement based on his local knowledge about the nearest environment limited by his or her field of view, trying to avoid collisions with other people, walls and obstacles and minimize uncertainty.

### 2.2. Interaction levels

Similar to Helbing's social forces model let $r_\alpha(t)$ be the location of an individual $\alpha$. The change of the current position with time is given by the equation:

$$\frac{dr_\alpha(t)}{dt} = v_\alpha(t). \tag{1}$$

Denote $F_\alpha(t)$ the sum of physical, social and psychological forces influencing each individual $\alpha$ and $\xi_\alpha(t)$ — random (or caused by factors not included in the model) fluctuations of the individual behavior, then:

$$\frac{dv_\alpha}{dt} = F_\alpha(t) + \xi_\alpha(t). \tag{2}$$

#### 2.2.1. Physical space: target direction and obstacle avoidance

The target force may be directed to the nearest visible exit marker (sign, wide door), or actual exit. The location of actual exit is not always known beforehand.

Also is possible that the desired direction $e_0^\alpha$ moves from physical point to the individual. By this way we describe a leadership phenomena. In this case known phenomena caused by the emergence of a leader in the crowd can be represented.

Each individual $\alpha$ is moving to the direction $e_\alpha^0$ adapting the actual velocity $v_\alpha$ to the desired speed $v_\alpha^0$, $v_\alpha = v_\alpha^0 e_\alpha^0$, within a relaxation time $\tau_\alpha$. $F_\alpha(t)$ is then given by the equation:

$$F_\alpha(t) = \frac{1}{\tau_\alpha}(v_\alpha^0 e_\alpha^0 - v_\alpha) + \sum_{\beta \neq \alpha} F_{\alpha\beta}(t) + \sum_O F_{\alpha O}(t) + \sum_P F_{\alpha P}(t), \tag{3}$$

where $F_{\alpha\beta}(t)$ is a social distance function describing the social forces acting on the individual, $F_{\alpha O}(t)$ is repulsive forces describing the individual need to keep a safety distance to walls and obstacles $O$, $F_{\alpha P}(t)$ is a psychological field force in the part of psychological map $P$.

Individuals keep a safety distance from walls and obstacles. The closer he or she is to them, the more uncomfortable the individual feels and the more intensive a repulsive force $F_{\alpha O}(t)$. This force decreases monotonically with the distance $\|r_\alpha - r_O^\alpha\|$ between the place of the individual $r_\alpha$ and the nearest point of the walls and obstacles. In the simplest case, this force can be expressed by the following equation in terms of a repulsive potential $V_O$:

$$F_{\alpha O}(t) = F_O(r_\alpha(t)) = -\nabla_{r_\alpha} V_O(\|r_\alpha - r_O^\alpha\|). \tag{4}$$

#### 2.2.2. Social interactions: constructing the social distance function

As we have already noted, at the social level interpersonal interaction is determined by many social and socio-psychological factors such as social norms, cultural features, social boundaries, social relations etc.

Thus, the physical distance between people during movement is different and depends on social features as well as the requirement to avoid collisions and provide a free space for movement. On close range, people will attempt to keep distance between each other; but on longer ranges, they will try to come closer, to seek support and look for exit together. The grouping tendency is enhancing in the situations of uncertainty. An illustration of this fact is the herding behavior and movement along the walls in the situation of poor visibility or disorientation in the absence of guidelines or contradictory impulses to action.

For modeling this force we propose to use the function similar to the Lennard-Jones potential directed from the individual $\alpha$ to $\beta$ (Fig.1). Its advantage is that it can account for both "attractive" and "repulsive" forces in Helbing model:

$$F_{\alpha\beta}(t) = -\nabla_{d_{\alpha\beta}} \epsilon_{\alpha\beta} \left[ \left( \frac{\sigma_{\alpha\beta}}{\|r_\alpha - r_\beta\|} \right)^{12} - \left( \frac{\sigma_{\alpha\beta}}{\|r_\alpha - r_\beta\|} \right)^{6} \right], \tag{5}$$

where $\|r_\alpha - r_\beta\|$ is the distance between $\alpha$ and $\beta$, $\epsilon_{\alpha\beta}$ is the depth of potential well (max attraction force), and $\sigma_{\alpha\beta}$ is the distance where the attraction becomes zero.

For homogeneous populations $\epsilon_{\alpha\beta} = \epsilon$ and $\sigma_{\alpha\beta} = \sigma$. In general case $\epsilon_{\alpha\beta} \neq \epsilon_{\beta\alpha}$ and $\sigma_{\alpha\beta} \neq \sigma_{\beta\alpha}$.

The Lennard-Jones potential can also be approximated by Buckingham's function with the exponential repulsive part, that could be more convenient to implement due to the relative computational efficiency:

$$F_{\alpha\beta}(t) = -\nabla_{d_{\alpha\beta}} \tilde{\epsilon}_{\alpha\beta} \left[ \exp \frac{\|r_\alpha - r_\beta\|}{\tilde{\sigma}_{\alpha\beta}} - \left( \frac{\tilde{\sigma}_{\alpha\beta}}{\|r_\alpha - r_\beta\|} \right)^{6} \right]. \tag{6}$$

Social distance function is computed for each pair of individuals in their field of visibility. The gradient of this potential creates a “social force” that can be attractive or repulsive depending on the distance between individuals. Intensity of this force varies according to the social relationships between them. In particular, attraction force is more intensive between familiar people, members of a group or family.

The proposed function allow us to take into account both of the opposite trends in social behavior observed during the motion — keeping a comfort distance and grouping.

#### 2.2.3. Psychological field: mapping the motion space

The individual moving trajectory is determined not only by the physical and social forces acting on the person, but also his/her psychological evaluation of the situation and surroundings.

Having cognitive abilities, an individual is able to estimate possible trajectories of movement and to choose subjectively optimal from them. Thus people precept

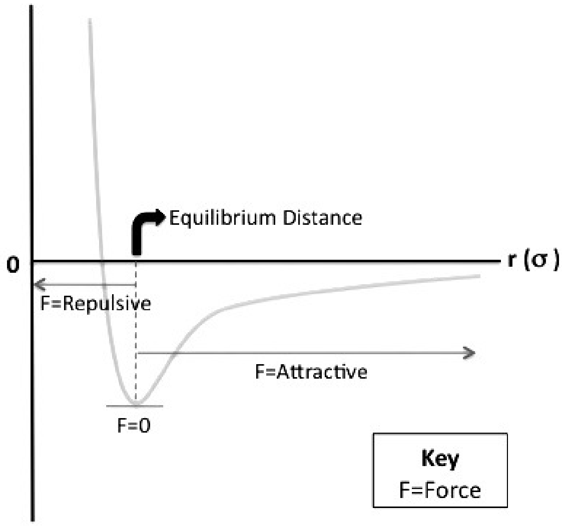


Figure 1. Social distance function

the surrounding area and create individual cognitive field maps. Each map includes areas with more or less comfort zones. The person choosing the trajectory tends to minimize the psychological costs, e.g. moving to the most psychologically beneficial parts of the field. Note that this path is not always the shortest or less time expensive [7].

It is also important to note that the behavior of an individual can be both field-dependent and field-independent, determined by cognitive personality, emotional state and specifics of the situation [8]. People who exhibit field-dependence tend to rely on outer information in decision making; field-independent people tend to be more autonomous, that may be a significant factor in determining individual movement strategies.

The individual psychological mapping has a pronounced specificity in a situation of certainty and uncertainty. In last case people are more likely to take emotional decisions; they show field-dependence behavior (including socio-dependent).

How to systematize the cognitive field map of different people? It is clear that the mapping is individual, but some common features to all or certain categories of participants exist. For example, a dark space will be less attractive than illuminated, the familiar path has more priority then the unknown, guide signs and signals may also mark parts of field as a more or less favorable to the movement. Furthermore, the occurrence of obstacles (e.g., the collapse of building structures) can mark the surrounding area as dangerous and costly for movement. Maps of people with different disabilities, such as blind or injured, or disoriented people (for example in the poor visibility situation) will also have similarities — they will seek a support and move along the walls or in the company of other individuals, and avoid open space, i.e. areas near walls will be marked as more suitable for further movement.

Psychological field map can be represented as a landscape with different heights or areas with greater or lesser density, which can be formalized in terms of the social-forces model as additional forces acting on the individual. In discomfort (dangerous) areas the force $F_{\alpha P}(t)$ directs toward the more psychologically comfortable (safe) zone will acts on each individual. In the individual field of view, there are may be several such zones, and, assessing a cost function for each possible path to the current time, we describe a process for forecasting and planning for each person.

Such zones can be either homogeneous or heterogeneous, depending on the situational context and structure of the motion space. For example, a psychological risk evaluation is not uniform near the fire, building collapse or radioactive contamination zone and practical uniform close to walls in the situation with low visibility conditions. In the last case the walls act as orientation points.

So, the psychological field function can be taken as:

$$F_{\alpha P}(t) = \begin{cases} C_P, & P \text{ is homogeneous,} \\ -\nabla_{r_\alpha} V_{p_{\max}}(\|r_\alpha - r^\alpha_{p_{\max}}\|), & P \text{ has } p_{\max}, \end{cases} \qquad (7)$$

where $C_P$ is a constant, different for different parts of field, $V_{p_{\max}}$ is a repulsive potential from the maximal discomfort point $p_{\max}$.

## 3. Simulations

According to the model we have created a Mathematical simulation for investigation the pedestrian flow dynamics in some simple situations.

The simulation environment is constructed as follows. We start with the rectangle region delimiting the evacuation space. Within this region, the following objects are set up:

- People — individuals trying to escape the space, modeled as particles with specific constraints. People are the only moving objects in the model;
- Walls and obstacles — rectangular impassable regions;
- Exits — the target regions where people should arrive, completing the evacuation. When a person crosses the exit boundary, she disappears from the model and its calculations;
- Special regions — the delimited areas where psychological forces and/or movement constraints are added.

The simulation advances in discrete timeframes. Within each timeframe, the following sequence of operations is performed:

- For each person in the model, the total force acting on him is calculated. It is a vector sum of all specific forces, which are:
  - Repulsive forces of walls and obstacles. In some contexts, walls can use Lennard-Johnes potential, but most commonly, exponential potential is applied;
  - Attractive force of the exit region. This force is constant; if there are multiple exits, the force operates on the nearest one.

  - Lehnard-John potentials of all other individuals in the model. It works as the attractive force if the individual considered is far enough, and the repulsive force if she is nearby. The individuals can be arranged in "groups" — where the attractive part of the force between individuals in different groups is omitted, which enables gathering effects. To avoid extremely strong forces, the scale of these vectors is capped at some pre-selected level.
  - Special area forces — e.g. the psychological forces of uncomfortable environment, leading away from the region;
  - Fluctuations — a small force with random direction and random magnitude is applied to each individual, to account for inevitable random variations in behavior.
- The total force is used to calculate acceleration applied to each individual, using the Newton's second law;
- Using Velocity Verlet integration algorithm, the acceleration vectors are integrated to velocities vectors;
- Human movement constraints are applied to all velocity vectors (all velocities are anisotropically rescaled and/or capped according to physical capabilities of individuals and movement direction — e.g. people move much faster in forward rather than backward directions.)
- Velocities are then integrated (again with Velocity Verlet) to the new positions of all people;
- Collisions are processed. If a person is too close to another person or to a wall or an obstacle, he is moved back to the minimum distance, and their velocities are modified according to the equation for two-dimensional perfect inelastic collision (walls and obstacles are not moving, of course).

This simple process can account for many peculiarities of human movement, enabling rich behavioral patterns observed in real evacuation experiments. We consider the following simulation set-ups:

1. A square room with the single exit and an obstacle;
2. A square room with the central area of poor visibility and the single exit;
3. A rectangular room with two exits and two randomly assigned groups;
4. A rectangular room with two exits on the opposite sides, a narrow pass and two opposite streams of evacuating persons.

Initially (at time $t = 0$) all pedestrians stand at some random location in the room and start to move in order to escape from the room according to the implemented rules.

### 3.1. Egress from the room with the single exit and obstacle

Even with this simplest configuration, complex interaction phenomenas can be observed, described in Helbing's papers and elsewhere. Among them are following:

- Herding and ignorance of available exits. If people are not sure what is the best thing to do, there is a tendency to show a "herding behavior", i. e. to

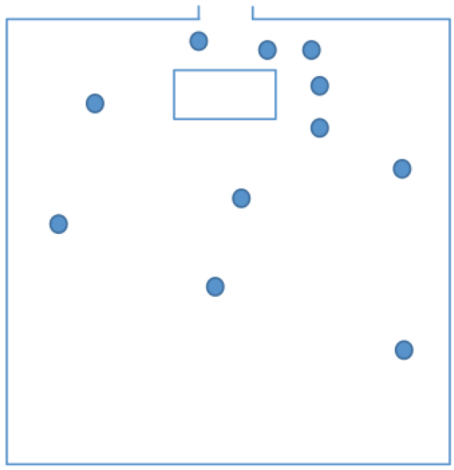

Figure 2. Egress the room with the single exit and obstacle

imitate the behavior of others. This effect is modeled by attraction factor in Lennard-Johnes potential (and, specifically, the relation of resulting gradient force to target force). When the target-directed force is weak enough, pedestrians observably gather in herd-like structures.

- “Freezing by heating”. At sufficiently high pedestrian densities, lanes are destroyed by increasing the fluctuation strength: "a solid state" is formed. It is characterized by an at least temporarily blocked, “frozen” situation so that one calls this paradoxical transition “freezing by heating”. As we use the capped version of Lennard-Johnes potential, the repulsive force might be not enough to stop people from stumbling upon each other, and the "solid state" of densely packed crowd becomes clearly visible. It is the most prominent in the near-exit area, where an obstacle blocks direct movement.

### 3.2. Egress from the room with poor visibility

We investigated the pedestrian behavior in the situation of evacuation from a room with poor visibility with different initial conditions.

We assumed that the zones close to the walls was more psychological comfortable (safe) for movement that all another space, so in conditions of poor visibility, pedestrians prefer to move along the walls to the nearest exit that widely reported in numerous research: the trajectories are concentrated along the walls, which are the support of the search output (for example, [9, 10]).

As results of simulation we obtained this characteristic pedestrians behavior and according to our results, we can say that Lennard-Johnes potential enables this and other predicted effects much better than the exponential repulsive potential in the base Helbing’s model.

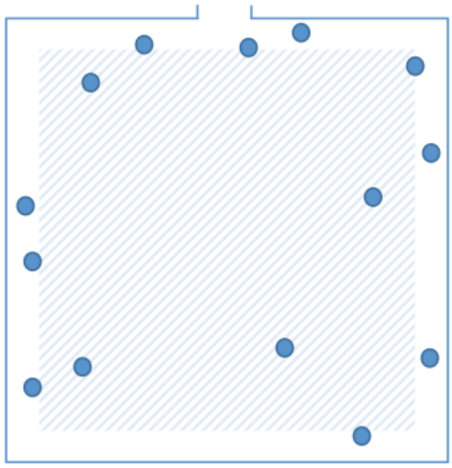

Figure 3. Egress the room with poor visibility

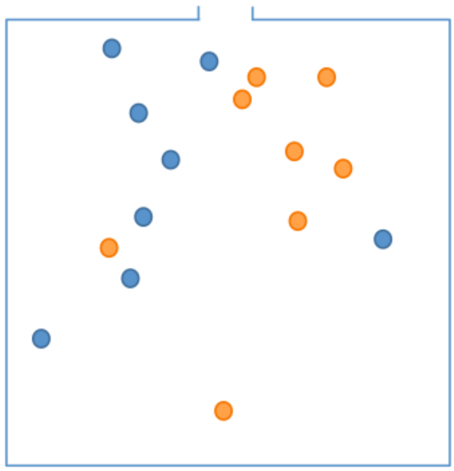

Figure 4. Egress from the room with two assigned groups

### 3.3. Egress from the room with two assigned groups

While in this configuration the evacuating individuals are randomly and uniformly distributed across the room, each of them is randomly assigned to one of the two groups — and the members of the same group are attracted to each other with much stronger force than to people in other groups (which is implemented by changing the coefficient in the attraction term in Lennard-Johnes potential). This simple arrangement leads to pre-grouping — the new effect that is not predicted in Helbing models, but easily observable in real experiments.

### 3.4. Two pedestrian streams moving in opposite directions

In this configuration we observe the lane formation: forming lanes of uniform walking direction. When the distance between pedestrians became too high, the

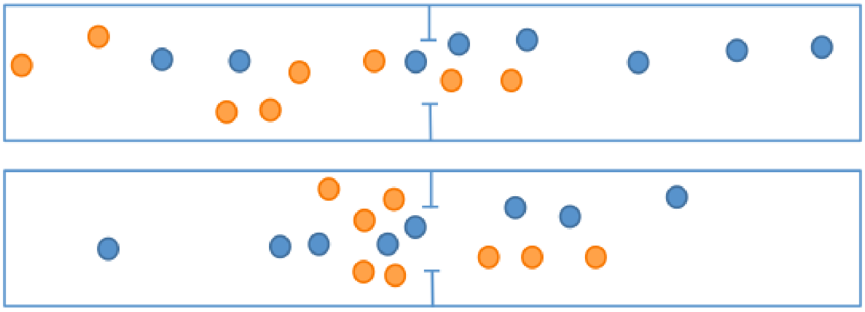

Figure 5. Two pedestrian streams moving in opposite directions

force of attraction is diminished. This leads to formation of several lanes moving to the target direction, where the social force of attraction applies only within a lane.

Also, we obtained the oscillatory flows predicted by Helbing's model. The structure of Lennard-Johnes potential combined with inertia naturally leads to oscillatory flow, as the potential gradient easily changes sign after crossing the repulsion-attraction barrier.

## 4. Results

The proposed model reflects the general properties of crowd behavior and takes into account individual characteristics of participants of movement, even though a quite simple representation of social forces with Lennard-Johnes potential is used.

The model is flexible and it able to be adapted to different situations (construction of physical space, specifics of participants — their physical and psychological conditions, age and others characteristics).

The extensibility of the model makes it possible to incorporate new factors and to accommodate new features with additional constraints and capping rules.

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# ПРИМЕНЕНИЕ ПОТЕНЦИАЛА ЛЕННАРДА-ДЖОНСА ДЛЯ ПРЕДСТАВЛЕНИЯ СОЦИАЛЬНЫХ СИЛ В МОДЕЛИ ХЕЛБИНГА ДЛЯ МОДЕЛИРОВАНИЯ ТОЛП

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**Аннотация.** В настоящей статье содержится краткий обзор различных методов моделирования поведения толп. В рамках моделей толпы, основанных на поведении частиц, рассматривается использование потенциала Леннарда-Джонса в качестве основной «социальной силы», определяющей взаимодействие между индивидуумами. Выполненная с помощью программного пакета Mathematica симуляция подтверждает для этого подхода наличие сложных эмергентных феноменов поведения толпы, описанных в модели Хелбинга и другими авторами.

**Ключевые слова:** поведение толпы, эвакуация, модель социальных сил, модель Хелбинга, потенциал Леннарда-Джонса.